

DELIGNE - ILLUSIE ET APPLICATIONS

INTRODUCTION
K-CORPS, CAR(K) = P.

$f \rightarrow X$ MORPHISMS OF K -VANISHING'S.

EST RELATABLES SUR $w_2 (|k| = w_2)$ SI

$\exists \quad Y_{w_2} \rightarrow X_{w_2}$ (X, Y PLATSSUR w_2)

POUR QUD. $(Y_{w_2} \xrightarrow{\sim} K \rightarrow X_{w_2}) \times_K$

THM(P) $U \subseteq X$ IMMERSION OUverts DE
 K -VANISHING'S CISSO RELATABLES SUR w_2 .

X PROPR. ALORS

$$\text{DIM}_K \left(M \left(H_{\Omega^1}^m(x) \rightarrow H_{\Omega^1}^m(y) \right) \right)$$

$$\text{DIM}_K \left(H^0(x, \Omega^m) \right)$$

$$\text{SI } 0 \leq m < p-1 \quad \text{②}$$

INTUITION:

LEMME:

i: $U \subseteq X$ VAR. CLASSES SUR UNES CAMES.

$$H^0(X, \mathcal{O}_X^i) \hookrightarrow H^0(U, \mathcal{O}_U^i)$$

PAR

IL SUFFIT $\mathcal{O}_X^i \hookrightarrow \mathcal{O}_U^i$

ON POUR SUPPOSER $X = \text{SPEC}(R_f)$ INTUIT

$$U = \text{SPEC}(R_h) \quad h \in R_{\leq i}$$

$$\mathcal{O}_R^i \hookrightarrow \mathcal{O}_{R_h}^i = \mathcal{O}_R^i \otimes R_h$$

$R \hookrightarrow R_f$ ET \mathcal{O}_R^i EST PUR

REAPPRE

THM (D-I) \times PROPV_K DÉFINISSES

ATMS $P \geq 0m(x)$ LAS V, D

$E^i \subseteq H^0(X, \Omega^{\otimes i}) \Rightarrow H^0_{\text{an}}(X) \text{ EST DIFFERENTIABLE.}$

CÔTA NEUTRE PAS ! UNIFORME

$$H^0(X, \Omega^i) \hookrightarrow H^0_{\text{an}}(X)$$

$$H^0_{\text{an}} = \frac{\ker(H^0(U, \Omega^i) \xrightarrow{\delta} H^0(U, \Omega^{i+1}))}{\text{im}(H^0_V \xrightarrow{\delta} H^0_{U,V})}$$

ESTANT DONC DES FORMES DIFFÉRENTIELLES

OUFS-OUFS-QUES QU'EST CE QU'IL'EST

QUI ?

LA QUESTION ESSENTIELLE, $U \subseteq \mathbb{K} \subseteq X$

$$H^0_{\mathcal{W}}(\mathcal{U}^{i-1}) \rightarrow H^0_{\mathcal{W}}(\mathcal{U}^i) \rightarrow H^0_{\mathcal{W}}(\mathcal{U}^{i+1})$$

$$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ H^0_{\mathcal{V}}(\mathcal{U}^{i-1}) \rightarrow H^0_{\mathcal{V}}(\mathcal{U}^i) \rightarrow H^0_{\mathcal{V}}(\mathcal{U}^{i+1})$$

Ex $i = 2$ $x = \mathbb{P}^1 \times \mathbb{P}^1$

$$\rho_{1C}(x) = 2 \times 21$$

$$C \subseteq x \rightsquigarrow (1,1) \in \rho_{1C}(x)$$

$$D \subseteq x \rightsquigarrow (1,2)$$

$$w = \mathbb{P}^1 \times \mathbb{P}^1 - C \leftarrow \text{AFF}_\text{red}$$

$$Y = W - D \leftarrow \text{AFF}_\text{red}.$$

$$kM \xrightarrow{\quad} H^2(x) = \emptyset \quad \#^2 = \emptyset < c, 0 >$$

$$H^0(\mathcal{O}) \xrightarrow{\quad} H^2_M(w) = \emptyset \leq \emptyset < 0 >$$

$$H^2_M(v) = 0$$

- CAMS COMPLEXOS.

$U \subseteq X$ \hookleftarrow PROPR. IMM. QUANT VAR SUR \mathbb{C} .

$$\dim_{\mathbb{C}} \left(\cap_{M'} H^m_{M'}(X) \rightarrow H^m_{M'}(\partial) \right) \geq$$

$$\dim_{\mathbb{C}} (H^0(X, \Omega^n))$$

PROOF (SI $X - U$ EST LISE)

SURF D'ORD CLEAV + THEOREME DE
HOMO MIXT.

$$H^{n-2c(-c)}_{M'}(X-U) \xrightarrow{\text{SIT.H.}} H^m_M(X) \rightarrow H^m_M(U)$$

$$\bigoplus_{P \geq 0} H^{P, n-P}$$

$$\oplus H$$

$$H^{n-2c(-c)}(X-U) \supseteq \bigoplus_{P \geq 0} H^{P, n-2c-P} \\ (-c) \quad \bigoplus_{P \geq 0} H^{P+2c, n-p}$$

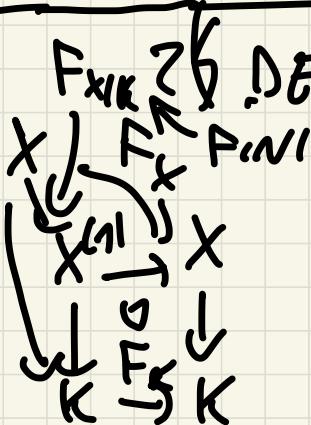
$$M(H_{\mathcal{X}^n}^{n,s}) \subseteq H^{n,s}$$

$$p+2s > 0 \quad M \cap H^{0,m} = 0$$

$$p=m$$

$$H^{n+2s, 0} = 0$$

$$n+2s > m-2s.$$



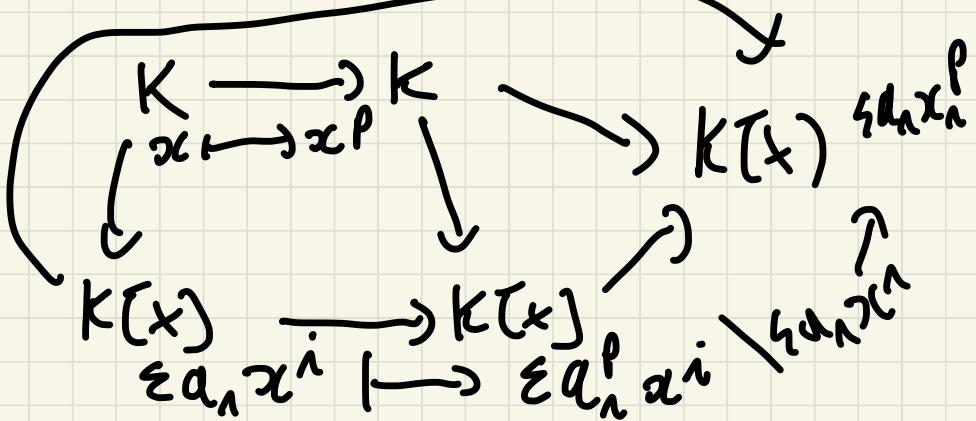
$F_{X/K} \geqslant$ DE (GAG - ILLUSIG).

$\text{CAN}(K) = p > 0$
 X/K (ISSG (AUGS/ QUDOR))

Ex

$$x = A^1$$

$$q d_n x^i \mapsto \varepsilon d_n x^i p$$



$$F_{x/K, \rho} \mathbb{U}_X := \left[F_{\mathcal{O}_X} \xrightarrow{F.d} F_{\mathcal{O}_X} \mathbb{U}^1 \rightarrow F_{\mathcal{O}_X} \mathbb{U}^2 \right]$$

MIRACLE: $F_{x/K, \rho}$ EST LINÉAIRE.

$$\mathbb{U}(x \cdot y^\rho) = \mathbb{U}(x)y^\rho + \underbrace{\rho \mathbb{U}(y^{\rho-1})y}_{=0}$$

RHM (CANNON)

(L3!) HOMOMORPHISME D'ALGEBRES

$$C^{-1}: \bigoplus_i \mathbb{U}_{X^{(1)}/K}^1 \xrightarrow{\sim} \bigoplus_i \mathcal{H}^i(F_{\mathcal{O}_X} X/K)$$

$$\text{TOUR QUIT} \quad C^{-1}(x_{(1)}) = x^{p-1} \mathbb{U}_X$$

$$\text{PROOF (SI } x = 1A^n \text{)}$$

$$\frac{\text{Ker}(F.d_{X/K}^i \rightarrow F.d_{X/K}^{i+1})}{\text{IM}(F.d_{X/K}^{i-1} \rightarrow F.d_{X/K}^i)}$$

$$\begin{array}{ll} i=0 & K[x] \\ i=1 & K(x) \mathbb{U}_X \end{array} \Bigg|$$

$$K[x] \ x \mapsto x^p$$

$$0 \rightarrow K[x^p] \rightarrow K[x] \xrightarrow{d} K[x] \xrightarrow{\sim} \underbrace{K[x]}_{\begin{array}{l} \text{---} \\ \text{---} \\ x \mapsto x^{p^n} \end{array}} \oplus K \cdot x^{p^n-1} dx$$

para $n=0$

$$0_{x(0)} = \mathcal{U}_{x^{(0)}} / K \cong \ker(F_{x^{(0)}} \rightarrow \mathcal{U}_x^1)$$

para $n=1$

$$\mathcal{U}_{x^{(1)}} / K \cong \frac{\ker(F_{x^{(1)}} \rightarrow \mathcal{U}_x^1)}{\ker(F_x^0 \rightarrow \mathcal{U}_x^1)}$$

THM (D-I) A $\cong (W_2(k))$

ROLEVANT X EST ASSOCIE
CARACTQUEMENR UN ISOMORPHIS

$$S_x \cdot \bigoplus_{n < p} \mathcal{U}_{x/K}^n [-i] \xrightarrow{\sim} Y_{\leq p} F_* \mathcal{U}_{x/K}^i$$

DANS $D(X^{(1)}, \mathcal{O}_{X^{(1)}})$

TEL QUE $\mathcal{H}^i(S_x) = 0$

$$\mathcal{U}_{x/K}^i \rightarrow \mathcal{H}^i(F_{x/K} \mathcal{U}_{x/K}^i)$$

$$M[-i]^j = M^j - i$$

$$N_{L(\alpha i)} = (-1)^m \delta_L$$

$$\gamma_{\leq p}^i \gamma_{\leq p-1}^{-i} : L^i \text{ est par}$$

$$(\gamma_{\leq p-1}^i)(i) = \begin{cases} L^i & i < p-2 \\ \kappa_m(w) & i \leq p-1 \\ 0 & i > p-1 \end{cases}$$

$$L^n \rightarrow L^{n+1} \rightarrow \dots \rightarrow L^{p-2} \rightarrow \kappa_m(w)$$

$$H^i_{\leq p}(L) = H^i(L) \quad i \leq p-1$$

$\lambda = 1$

$\text{REN}(F, \Omega^2 \rightarrow \Omega^2)$

$\Omega^1_{X^{(1)}}/K$

$\sum_{i=1}^n$

$\text{ker } (\Omega^1_X \rightarrow \Omega^2_X)$

$\text{im } (\Omega^0 \rightarrow \Omega^0_K)$

$D - I \Rightarrow \text{TAU}$

$\lambda: V \subseteq X \rightsquigarrow \lambda: V_{w_1} \subseteq X_{w_1}$

$D - I \Rightarrow \bigoplus_{i < p} \Omega^i_{X^{(1)} / K} [(-i)] - \bigoplus_{i < p} \Omega^i_{V^{(1)} / K} [(-i)]$

$\downarrow \zeta \quad . \quad \downarrow \zeta$

$\bigoplus_{i < p} F_{X/K} \Omega^i_{X/K} \rightarrow \bigoplus_{i < p} F_{X/K} \Omega^i_{X/K}$

$$\lambda \leq p-1 \quad H^{\dot{\alpha}}(x^{(1)}, \Omega_{x^{(1)}/K}^1) \hookrightarrow H^{\dot{\alpha}}(\Omega_{x^{(1)}/K}^1)$$

|| . \hookrightarrow
LEMMA

$$H^{\dot{\alpha}}\left(x^{(1)}, \bigoplus_{\lambda < p} \Omega_{x^{(1)}/K}^1[-\lambda]\right) \rightarrow H^{\dot{\alpha}}\left(V^{(1)}, \bigoplus_{\lambda < p} \Omega_{V^{(1)}/K}^1[-\lambda]\right)$$

\hookrightarrow \hookrightarrow

$$H^{\dot{\alpha}}\left(x^{(1)}, \bigoplus_{\lambda < p} F_{x/K, \lambda} \Omega_{x/K}^1\right) \rightarrow H^{\dot{\alpha}}(X^{(1)}, \dots)$$

\hookrightarrow \hookrightarrow

$$H^{\dot{\alpha}}\left(x^{(1)}, F_{x/K, \lambda} \Omega_{x/K}^1\right) \rightarrow H^{\dot{\alpha}}(\dots)$$

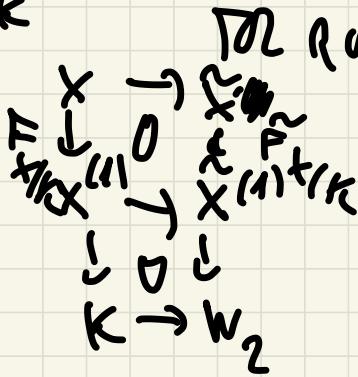
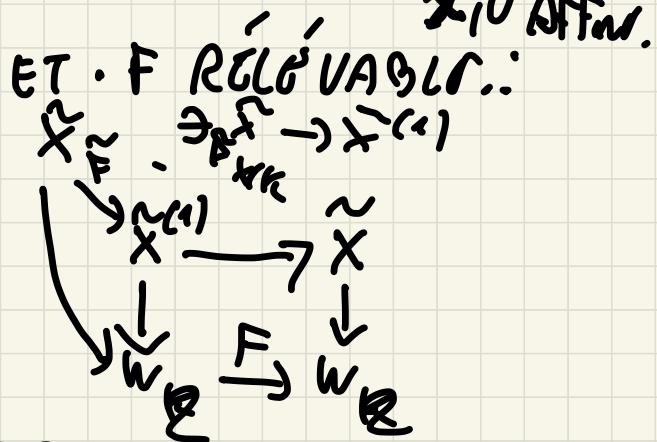
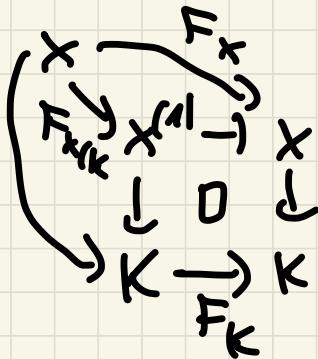
\hookrightarrow \hookrightarrow

$$H^{\dot{\alpha}}(x, \Omega_{x/K}^1) \rightarrow H^{\dot{\alpha}}_{\Omega_K}(x) \rightarrow H^{\dot{\alpha}}_{\Omega_K}(x)$$

|| \rightarrow \downarrow

③ PROUVEZ QU' $\Delta\Gamma$ EST UNICIEL.

POUR $\lambda = 1$



(LOCALISATION
UNIF)

on peut : — — —

$$n_{X^{(1)}/K}^1 \hookrightarrow \text{ker}(F_{\Omega^1} \rightarrow F_{\Omega^2})$$

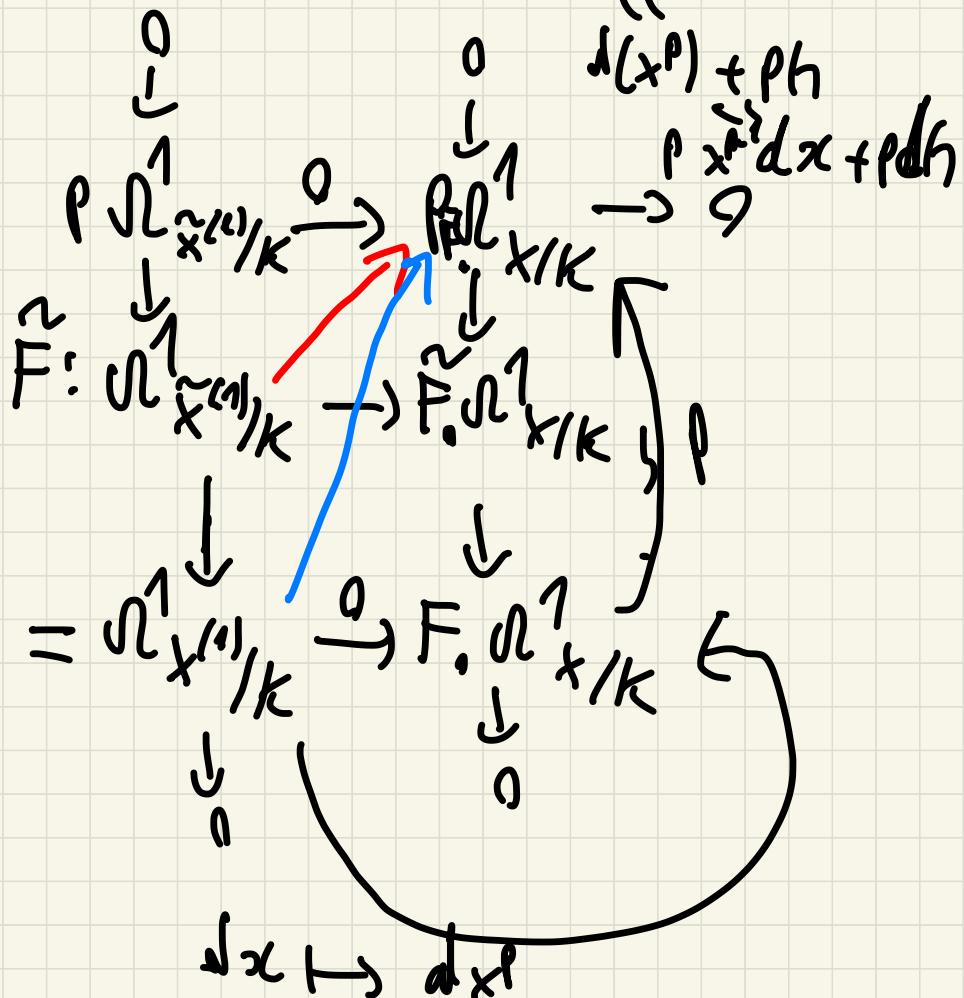
$$\stackrel{?}{\hookrightarrow} F_{\Omega^1_X} \leftarrow \tilde{F}_* n_{X^{(1)}}^1 \leftarrow P F_{\Omega^1_X} \hookrightarrow$$

$$\begin{aligned} ① \quad & F_{\Omega^1_X}^{(1)} \xrightarrow{\text{id}} F_{\Omega^1_X}^{(1)} \\ & K(x)^{1/x} \xrightarrow{x} K(x)^{1/x} \\ & x \mapsto \sqrt[x]{x} = \eta \end{aligned}$$

$$\xrightarrow{\quad \text{et } P \quad} Pw_1(K) - 1 w_2(K)$$

- $\text{Im}(\tilde{F}) \subseteq P\mathcal{U}_{x/K}^1$
 - $\tilde{F}(P\mathcal{U}_{\tilde{x}/K}^1) \subseteq P\text{Im}(\tilde{F}) \subseteq P^2\mathcal{U}_{x/K}^1$
- $\tilde{F}(x) = x^p + ph$

$$d(\tilde{F}(x)) = d(x^p + ph)$$



$$f: P^{-1} \tilde{F}. \quad \text{EST - CQ QCR - JF = ?}$$

$- X^q(f_h) = C^{-1}$

$$\tilde{F}(x_0 \oplus 1) = x^P + P h$$

$$h(x_0 \oplus 1) = x_0^P \sqrt{x_0} + dh(x)$$

$$P \sqrt{(x_0 \oplus 1)} = \tilde{F}(x_0 \oplus 1) = P x_0^{P-1} P \sqrt{|h|}$$

II

$$P x_0^{P-1} \sqrt{x} + P \sqrt{|h|}$$

$h = C^{-1}$

$$df(dx_0 \oplus 1) = 0$$

II

$$d P^{-1} \tilde{F}(dx_0 \oplus 1) = 0$$

$$0 \rightarrow \mathbb{F}_p \xrightarrow{\sim} \frac{\mathbb{F}_p[x]}{x^2} \rightarrow \mathbb{F}_p$$

$$\begin{aligned} K(x,y) dx \oplus K(x,y) dy \\ \downarrow \\ K(x,y) \sqrt{x} \wedge \sqrt{y} \end{aligned}$$

$$h(x,y) \sqrt{x} + g(x,y) \sqrt{y}$$

$$\downarrow \sqrt{ }$$

$$\underbrace{g(x,y)}_{\sqrt{x}} - \left. \frac{df(x,y)}{dy} \right|_{x,y} x \wedge y$$

$$g(x,y) = 0$$

$$df = \frac{\sqrt{f(x_0)}}{\sqrt{x}} \int_{x_0}^x \frac{f'(x)}{\sqrt{f(x)}} dx$$